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90. Proposed by **HARRY S. VANDIVER**, Bala, Pa.

Prove that it is always possible to find an infinite number of positive integral values of x, y, and z, such that the relation $z^2=x^2+bxy+cy^2$ is satisfied, b and c being any integers whatever.

Solution by the PROPOSER.

If $x^2+bxy+cy^2=z^2$....(1), then solving for x,

$$x=\frac{-b\pm\sqrt{[b^2y^2-4(cy^2-z^2)]}}{2}.$$

If x is to be a positive integer it follows that $b^2y^2-4cy^2+4z^2=$a square. So that the solution of the original problem is reduced to a solution of

$$X^2+AY^2=Z^2....(2).$$

It may be shown directly that this relation possesses an infinite number of integral solutions for X, Y, and Z, $-A$ being any integer whatever.

$$(m^2+An^2)^k=X^2+AY^2$$
$$\text{for } (m+n\sqrt{-A})^k=X+Y\sqrt{-A}$$
$$\text{and } (m-n\sqrt{-A})^k=X-Y\sqrt{-A}$$

whence, by multiplication, $m^2+An^2=X^2+AY^2$.

Hence we may assume in (2) that $Z=m^2+An^2$; m and n being any numbers. Then

$$(m^2+An^2)^2=X^2+AY^2$$
$$\text{and } (m+n\sqrt{-A})^2=X+Y\sqrt{-A}.$$

Expanding, we have $m^2-n^2A+2mn\sqrt{-A}=X+Y\sqrt{-A}$. Whence

$$X=m^2-n^2A$$
$$Y=2mn$$
$$\text{and by assumption, } Z=m^2+An^2.$$

Hence, (2) has an infinite number of solutions, since a solution is found for every value given to m and n. Also, for every solution of (2) there is a solution of (1), hence (1) has an infinite number of solutions, which was to be proved.

Proceeding in like manner, it can be shown that the relation

$$x^2+bxy+cy^2=z^n....(3)\ (n, \text{ an integer})$$

also possesses an indefinite number of solutions, but the method which has been used does not enable us to find *all* the solutions of either (1) or (3).